Spreading Processes in Multilayer Networks

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Abstract—Several systems can be modeled as sets of interconnected networks or networks with multiple types of connections, here generally called multilayer networks. Spreading processes such as information propagation among users of online social networks, or the diffusion of pathogens among individuals through their contact network, are fundamental phenomena occurring in these networks. However, while information diffusion in single networks has received considerable attention from various disciplines for over a decade, spreading processes in multilayer networks is still a young research area presenting many challenging research issues. In this paper, we review the main models, results and applications of multilayer spreading processes and discuss some promising research directions.

Index Terms—Multilayer network, multiplex, interconnected, spreading processes, information diffusion

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1 INTRODUCTION

MANY real-world systems can be modeled as networks, i.e., sets of interconnected entities. In some cases the connections between these entities represent communication channels: they indicate that information items present at one of the entities can be transferred, or propagated, to some neighbor entities. A typical example is represented by online social networks, where information can move from one user account to the other through, e.g., *friendship* or *following* connections, but several other scenarios exist where the nodes of the network are not human beings (e.g., computer networks and the so-called Internet of things) and the items traversing the network are not text messages but for instance viral agents, rumors, behaviors, pathogens or digital viruses. These are all examples of spreading processes.

Studying the diffusion of pathogens has a long history in biological systems, and a robust analytic framework has developed in epidemiology for modeling this type of spreading processes [1], [2]. With the advent of network science, the traditional epidemic models were extended to incorporate the structure of the underlying network [3] and utilized to study network epidemics [4], [5], [6], [7]. Such modeling has recently attracted considerable attention in spreading processes over communication systems [8], [9], [10] and online social communities [11], [12].

However, although spreading processes on networks have been thoroughly studied during over the last decade [13], real spreading phenomena are seldom constrained into a single network (called monoplex network). This is evident in online information propagation, where the process of switching network while sharing information on social media has become a basic functionality explicitly provided by many platforms. Another example is represented by the spreading of epidemics propagated by human beings traveling via multiple transport networks (airplanes, trains, etc.).

In this paper, we focus on the practically relevant topic of *spreading processes* in *multilayer networks* a generic term that we use to refer to a number of models involving multiple networks, called *interconnected networks* [14], or multiple types of relationships, called *multiplex networks* [15]. Multilayer networks are also known as interdependent [16], [17], [18], multidimensional [19], multiple [20], multisliced [21], multilevel [22], [23] networks, and networks of networks [24], [25]. Historically, networks were first inspected from the multilayer perspective by sociologists in works such as [26] in the late 1930s, and research continued in subsequent years [27], [28], [29], [30], [31].

Multilayer networks have attracted interest again in recent years; the reader is referred to some recent review articles and books [24], [32], [33] for general overviews of multilayer networks. Some of these works also contain discussions on spreading processes ([33, Section 5] and [32, Section 4.6]). Our paper is not intended to be a general review on multilayer networks; rather, we focus on spreading processes and therefore provide a more detailed coverage of this topic, also in terms of covered approaches, including a comprehensive categorization of models, applications and results that can help the reader to navigate the varied research landscape. We do not assume that the reader is mathematically inclined—although some technical details cannot be completely avoided and therefore we believe that this work can be of interest for a more general audience. However, we also recommend the interested reader to check the aforementioned references for an alternative presentation of the area.

In addition to spreading processes (which is the focus of this paper), numerous other types of diffusion processes on multilayer networks have been studied, including cascading failures [16], [34], [35], [36], cooperative behavior [37], [38], [39], [40], and synchronization [41], [42], [43].

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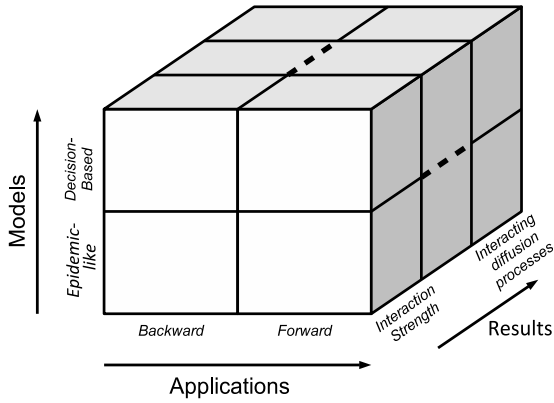
Fig. 1. Three main dimensions for analyzing spreading processes in multilayer networks: (i) how to model the spreading processes, (ii) what results we can obtain using these models and (iii) how these results can be exploited in real applications.

When only single networks are involved, it is well known that for all the processes above the structure of the network plays an important role on the outcomes of the process. For example, behavior spreading can stall when it enters a tightly-knit community within the network [44]. The same is true when multilayer networks are involved, but the effect of the layer structures and their interdependence may differ from the single-network case. Today, the study of spreading processes in multilayer networks is a young and rapidly evolving research area facing challenging issues. In this paper, we provide a homogeneous overview of current results on the effect of multiple layers and other network features on the spreading of different types of items, and identify unexplored areas.

To this end, we analyze the topic of spreading processes in multilayer networks according to three main aspects: (i) how spreading processes can be modeled (Section 3), (ii) what results can be obtained from these models (Section 4) and (iii) how these results can be exploited in real applications (Section 5). These aspects are summarized in Fig. 1. The paper follows the same structure: after introducing the basic concepts (multilayer networks, spreading processes in multilayer networks and variables used to study these phenomena) we devote one section to each of the aforementioned aspects. Finally we present a set of open problems in the area that in our opinion still require significant research efforts (Section 6).

2 PRELIMINARIES

In this section we introduce the concepts of multilayer network and spreading processes in multilayer networks, and the main methods and variables used to study these processes.

We assume that the reader is already familiar with the concept of *graph*: a graph $G = (V, E)$ is a finite set of nodes (vertices) V and a set of (ordered or unordered) pairs $E \subseteq V \times V$. A monoplex network is a (usually directed) graph. A multilayer network is a data structure made of multiple layers, where each layer is a monoplex network. Here we use the general mathematical framework defined in [32] (see also [45] as the first attempts to provide multilayer network science with a consistent mathematical representation). In this framework, the same nodes can appear in

multiple layers and nodes on different layers can be connected to each other. As an example, in Fig. 2a the pairs (v_4, l_2), (v_4, l_3) and (v_5, l_2) identify specific nodes in the different layers, in particular node v_4 on layers l_2 and l_3 and node v_5 on layer l_2. Layer l_2 corresponds to a monoplex network, with simple edges like $((v_4, l_2), (v_5, l_2))$—or just (v_4, v_5) if we know we are referring to layer l_2. In addition, we can have edges between layers, e.g., $((v_4, l_2), (v_4, l_3))$. In the context of this paper, edges model e.g. communication channels: in Fig. 2a if v_6 has some information on layer l_2 s/he can propagate it to v_5 on the same layer or send it to v_2 on layer l_3.

Building on this basic model several attributes can be added to nodes and edges. For example, we can introduce a temporal dimension and make a distinction between node v_4 on layer l_2 at time t_0 (v_4, l_2, t_0) and the same node at time t_1 (v_4, l_2, t_1), and we can then add edges among these extended nodes, like $((v_4, l_2, t_0), (v_4, l_2, t_1))$, or $((v_6, l_2, t_0), (v_2, l_3, t_0))$. In [32] these attributes (layer, time, etc.) are called *aspects*. The notation introduced in the remainder of the paper is summarized in Table 1.

Generally, we can consider two extreme cases for the nodes in a multilayer network. In one case all layers contain the same set of nodes, as in the case of individuals that may take part to different online social networks (i.e., layers) at the same time. A multilayer network where all layers contain exactly the same set of nodes is called *multiplex network* [15] (However, in *partially interconnected multiplex networks*, as defined in Section 4.2.3, only a fraction of the nodes are present in all layers). At the other extreme, each node of a multilayer network may belong to exactly one layer, resulting in a data structure sometimes called *interconnected* [14] (or *interdependent* [16], [17], [18]) network; in interconnected networks self-interactions across different layers are therefore not possible. In a different perspective, interconnected networks can be viewed as "interconnected communities within a single, larger network" [46]. As an example of interconnected networks we may consider the power and communication infrastructures, where the functionality of each one of the two networks depends on the other, and failure of particular nodes in either of the networks compromises the operation of the other network [16]. As another example, one can consider people from different countries as separate citizen networks connected through an air transportation network, i.e. airports with direct flights between them.

As said, connections between nodes on the same or different layers represent channels through which different types of items can propagate, giving rise to spreading processes. In general, spreading process can refer to the diffusion of pathogens, rumors, behaviors, or the coverage of a newsheadline in different newsgroups and weblogs. Although all the above contexts share some common aspects, there are specific features differentiating the various types of spreading processes. For example, in the case of spreading of some behavior in a community, people usually choose which behavior to adopt. On the other hand, in the case of epidemics there is no decision made by the individuals who are infected. These topics are thoroughly covered in Section 3. In the current section, we present the key concepts that may arise in the analysis of spreading processes.

(a) *A Multilayer Network*

(b) *A spreading process starting from seed node (v_4, l_2) on the multilayer network in Figure 2(a). This is also called a* cascade, *and we refer to it as C_1*

(c) *A cascade C_2 generated at seed node (v_4, l_1)*

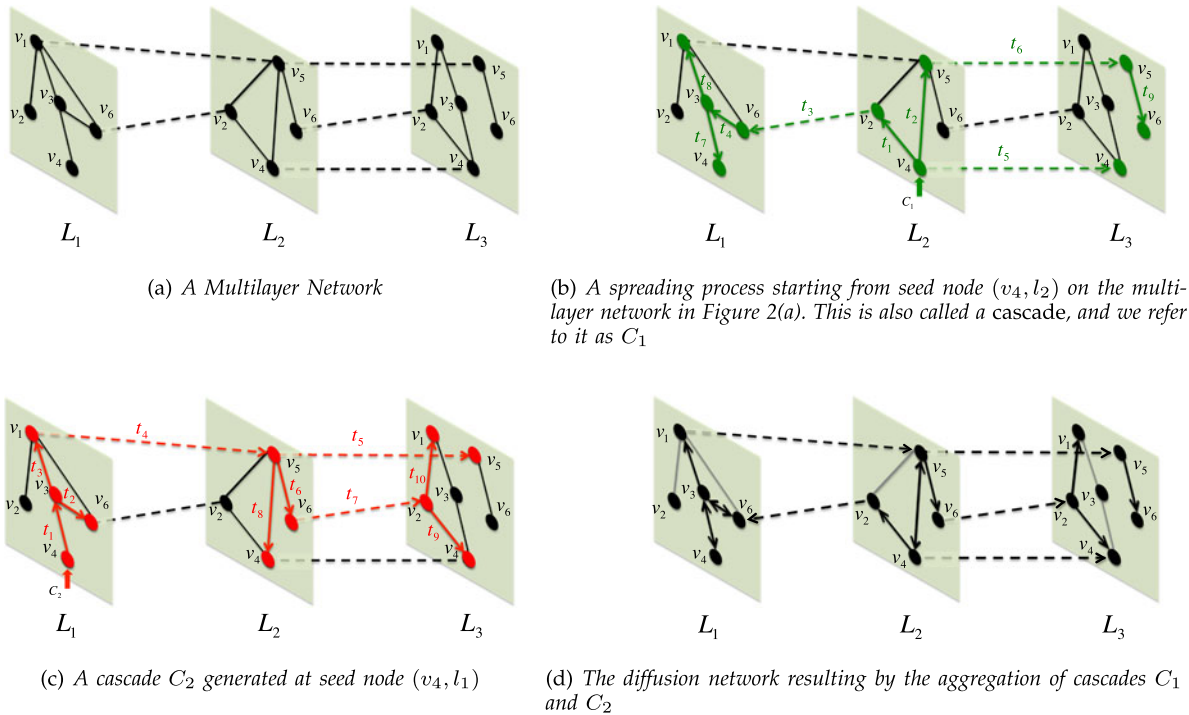(d) *The diffusion network resulting by the aggregation of cascades C_1 and C_2*

Fig. 2. Spreading processes in multilayer networks. **(a)** The underlying multilayer network. In this example edges are undirected and solid and dashed lines represent intra-layer and inter-layer edges, respectively. Notice that nodes have the same name across layers and may not be present in some of them. **(b,c)** Example cascades C_1 and C_2 diffusing over the underlying multilayer network starting from nodes (v_4, l_2) and (v_4, l_1), respectively. The arrows represent the direction of spreading processes: dashed arrows represent inter-layer information propagation, edges $((v_5, l_2), (v_5, l_3))$ and $((v_4, l_2), (v_4, l_3))$ in C_1 and edge $((v_5, l_2), (v_5, l_3))$ in C_2 are examples of *same-node inter-layer* diffusion, edge $((v_2, l_2), (v_6, l_1))$ in C_1 and edges $((v_1, l_1), (v_5, l_2))$ and $((v_6, l_2), (v_2, l_3))$ in C_2 are examples of *other-node inter-layer* diffusion. The other edges in C_1 and C_2 are examples of *other-node intra-layer* diffusion. Nodes do not need to participates in *same-node inter-layer* diffusion. For example, node v_4 in L_2, spreads the information to Layer L_3 but not to L_1. Also, node v_4 in L_1 does not spread to layers L_2 and L_3. **(d)** The subgraph resulting from the aggregation of cascades C_1 and C_2 forms the diffusion network.

The evidence left from the spreading of a particular piece of information over a monoplex network is called *(information) cascade* [47], [48]. This concept can be extended for multilayer networks [49], as shown in Figs. 2b and 2c. It also generates an implicit network as shown in Fig. 2d. Therefore we will sometimes distinguish between a *diffusion network* (i.e., the actual connections traversed during the spreading process) and an *underlying (multilayer) network*. A diffusion network is defined by the sequence of nodes traversed by a certain piece of information or other item. In a multilayer network a cascade can be represented as a set of tuples (u, l_u, v, l_v, t) where t represents the timestamp when the propagated item passed from node u in layer l_u to node v in layer l_v. We call *seed* the first node of the tuple with the minimum timestamp. While this is the minimum amount of information needed to meaningfully describe a spreading

process, specific models reviewed in Section 3 augment these tuples with additional parameters (i.e., a state space and a set of rules for state-transition) providing more details about the cascade.

One of the important ideas in the context of spreading processes in multilayer networks is the fact that items can also spread from one layer to another. In general there are four possibilities for an item to traverse a multilayer network (see Fig. 3): *same-node inter-layer*, when the cascade switches layer but remains on the same node, e.g., when a Facebook post is shared on Twitter by the author of the same post; *other-node inter-layer*, when a cascade continues spreading to another node in another layer, e.g., exchanging mails between users with different mail accounts (e.g., gmail and yahoo). In third type, *other-node intra-layer*, the cascade continues spreading through the same

TABLE 1
Notation

V	The set of nodes in a multilayer network
L	The set of layers in a multilayer network
n	The number of nodes in a multilayer network
(u, l_u)	Corresponds to node u on layer l_u in a multilayer network
$((u, l_u), (v, l_v))$	The tuple representing an edge between node u on layer l_u and node v on layer l_v in a multilayer network
C	An information cascade
$(u, l_u, v, l_v, t)_C$	The entries of the set denoted by the information cascade C
D	A (multilayer) diffusion network

	Same Node	Other Node
Same Layer	(unused)	The cascade spreads to a different node on the same layer
Other Layer	The cascade switches layer but remains on the same node	The cascade continues spreading on a different node on another layer

Fig. 3. Different possibilities for spreading an item from one layer to another in a multilayer network.

layer, e.g., retweeting a post in Twitter. It is worth noting that inter-layer spreading may involve a *layer-crossing overhead* (which is also called *layer-switching overhead*) [50]. The fourth combination, *same-node intra-layer*, is generally not considered meaningful and therefore omitted in all the spreading studies we have considered. Magnani and Rossi [51] introduced a model where the same individuals can have multiple nodes (e.g., accounts) on the same network. In this case, information might flow from one individual to the same individual, from one account to the other. However, to the best of our knowledge this model has not been used to study spreading processes yet.

Fig. 2 presents a summary of the concepts of underlying multilayer network, different types of information cascades and the resulting diffusion network. The corresponding terminology introduced so far is indicated in Table 2 for quick reference.

Studies based on spreading processes can be categorized into three types. *Empirical studies* involve the analysis of real datasets, either complete or sampled [52], [53]. These studies would be extremely important to understand the real dynamics of information diffusion. However, to the best of our knowledge there are so far no works based on real datasets of information diffusion in multilayer networks. Unlike cases involving a monoplex network [54], [55], [56], [57], [58], it is non-trivial to analyze the process in multilayer networks. In [51], [59] the authors use sampling methods for collecting data from multiple online social networks, but do not have access to information cascades. Given the difficulty in collecting real datasets including both the spreading process and the underlying network where diffusion takes

place, the totality of existing works on multilayer spreading are either *simulation-based studies*, where a synthetic [49], [50], [60], [61], [62], [63], [64] or real [61], [65], [66], [67], [68], [69], [70], [71] network is used to host artificial spreading processes, and *analytic studies* working with mathematical models of information diffusion [61], [62], [72], [73], [74].

Both simulation and analytic studies are based on the observation of the behavior of specific variables of interest depending on some input parameters. A basic parameter included in all studies is the *transmissibility probability*, indicating the probability of transmitting an item (i.e., transferring an infection, passing a message, spreading some rumor) from one node to the other [50]. Care should be taken in providing a unique definition of this concept, because variations can be found in the literature. For example, in [72] transmissibility is considered to be the mean value of this probability computed among all neighbor nodes, and [50], [72] distinguish between different kinds of transmissibility—homogeneous and heterogeneous. Other important input parameters in simulation studies are the *type of underlying networks* (e.g., random [62], scale-free [72], small-world [49], etc) and the *relationships between different layers* [14], [71], [73] (e.g., the correlation between node degrees [73]).

We conclude this section by presenting the main dependent variables used in different spreading studies. The so-called *epidemic threshold* [1], [75] is one of the key observations in epidemic-like models (refer to Section 3), and indicates a value of transmissibility above which the diffusion involves the whole (or most of the) network, e.g., the diffusion network is a giant component of the underlying network. It is known that in monoplex networks the value of the epidemic threshold is closely related to the largest eigenvalue of the network's adjacency matrix [76], [77]. Furthermore, recent work suggests that the epidemic threshold in a multiplex network cannot be larger than the epidemic thresholds of individual layers [50]. In the context of interacting spreading processes in multilayer networks (refer to Section 4.2.6), two types of thresholds have recently been introduced, called *survival threshold* and *absolute-dominance threshold*: they measure if a spreading process will survive and whether it can completely remove another competing process [78]. Another dependent variable is the *infection size*, generally defined as the number or fraction of nodes in the

TABLE 2
Basic Terminology

Term	Explanation
Multilayer Network	General term for a network with multiple layers
Multiplex Network	Multilayer network with exactly the same set of nodes across all layers
Interconnected Network	Multilayer network in which the nodes are of different types
Monoplex Network	Network with a single layer
Information Cascade	The trace left by the spread of information
Seed(s)	The node(s) from which an information cascade starts spreading
Diffusion Network	The subgraph resulted from the aggregation of covered subgraphs of information cascades
Intra/Inter-Layer Diffusion	The spread of an information cascade within/between layers of a multilayer network

TABLE 3
The Main Dependent Variables Used in Different Studies on Spreading Processes

Type	Variable Name	Definition
Static	Transmissibility	The probability of transmitting an item from one node to another.
	Epidemic Threshold	A value of transmissibility above which the spreading process involves most of the network.
	Survival Threshold	Given two interacting spreading processes, the survival threshold is a critical point for effective infection rate of one process above which this process survives [78].
	Absolute-dominance Threshold	Given two interacting spreading processes, the absolute-dominance threshold is a critical point for effective infection rate of the first process such that not only this process survives but also it removes the competing process [78].
	Infection Size	The number or fraction of nodes in the diffusion network.
	Cascade Size	The number of infected nodes in a cascade.
	Infection Rate	The average rate of being in contact over a link.
Temporal	Epidemic Dynamics	The percentage of infected nodes (i.e., infection size) at a specific time.
	Cascade Velocity	How fast an item reaches some relevant nodes or a given number of nodes in a cascade.
	Survival Probability	The probability of an infection started from a single infected node being active at a specific time.
Target-based	Outbreak Probability	The chance that a seed infection gives rise to an epidemic outbreak.
	Recall	The ratio of the number of relevant nodes in the diffusion network divided by the total number of relevant nodes.
	Precision	The ratio of number of relevant nodes in the diffusion network divided by the total number of nodes in the diffusion network.

diffusion network, i.e., those reached by the spreading process. Similar terms such as *outbreak size* [75] or *cascade size* [49] have also been used in the literature to refer to this quantity. The *Infection rate*, representing the average rate of being in contact over a link, is also a frequently studied dependent variable.

While epidemic threshold and infection size are static measures of a spreading process, some observational variables also take temporal aspects into account. For example, in some epidemic models an infected node may recover from the disease or may die and be removed from the network. As a consequence, the number of "infected" nodes changes with time. The percentage of infected nodes (i.e., infection size) at a specific time is sometimes called *epidemic dynamics* [79]; the *cascade velocity* measures how fast an item (e.g., a message) reaches some relevant nodes or a given number of nodes in a cascade [49]. Finally, we can study the *survival probability*, which is the probability that an infection, started from a single node, is still active at time t [46]. *Outbreak probability* indicates the probability that a seed infection gives rise to an epidemic outbreak [50], [80].

Recall and *precision* are two widely used measures in the field of information retrieval and pattern recognition. For spreading phenomena over networks, recall can be defined as the ratio of the number of relevant nodes in the diffusion network divided by the total number of relevant nodes, while precision is the ratio of the number of relevant nodes in the diffusion network divided by the total number of nodes in the diffusion network [81]. In this context, relevance is an application-specific measure of "interest" of a node in the item that it is being spread.

Table 3 summarizes the definitions of the aforementioned variables.

3 MODELING SPREADING PROCESSES IN MULTILAYER NETWORKS

As said, it is non-trivial to obtain real data for analyzing spreading processes in multilayer networks. Therefore, as an alternative approach, modeling can be used for understanding and analyzing the dynamics of spreading processes over the networks. Here, we discuss various research works which have attempted to model spreading processes in multilayer networks. We first review and categorize existing spreading models (Section 3.1) and then describe theoretical approaches for the analysis of these models (Section 3.2).

3.1 Review and Classification of Existing Spreading Models

We categorize existing models in two groups: epidemic-like (Section 3.1.1) and decision-based (Section 3.1.2).

3.1.1 Epidemic-Like Models

In epidemic-like models, generally used for modeling disease and influence spreading, the probability that a node becomes infected by a spreading process (e.g., disease spreading) is determined by its neighbors or adjacent nodes [3]. Most of the work on modeling the dynamics of spreading over multilayer networks has used epidemic models such as SIR [46], [50], [61], [62], [63], [64], [72], [82], SIS [14], [73], [74] and SI1I2R [68], [69].

The dynamics of epidemic spreading according to the SIR and SIS models are described as a three and two-state process, respectively. The spreading process starts with an initial infected set of nodes, called seeds. An infected node diffuses the infection (i.e., information, disease) to a susceptible neighbor with infection rate β. The infected nodes can recover after time τ from the moment of infection, as in the

susceptible-infected-recovered (SIR) model; or they can change their state back to susceptible as in the susceptible-infected-susceptible (SIS) model. Many extensions have been applied to SIR and SIS models; interested readers can refer to [83], [84] for more details and various extensions. As one of the most important extensions, Goldenberg et al. [85] proposed a discrete-time version of the SIR model called *Independent Cascade Model (ICM)*, where time proceeds in discrete time steps. In this model, each infected node u at time t can infect each of its neighbors. If the infection succeeds, then neighbor v will become infected at step $t + 1$. ICM is often used in the literature on influence spreading. In [70], the authors extended this model to analyze the dynamics of multiple cascades over a multiplex network.

In a monoplex network, the probability of transferring an (information) item from one node to another (i.e., transmissibility) is computed as $T = 1 - e^{\lambda}$ in the continuous case [5], where λ is the effective infection rate. Also, $\lambda = \beta\tau$, where β is the infection rate and τ represents the time for which a node remains infected [5]. In the case of multilayer networks, the infection may diffuse over inter- and intra-layer connections at different speeds, meaning that we have different infection rates (i.e., transmissibilities) across the links of each layer and also the links between the layers. Therefore, most of the works on spreading processes over multilayer networks [14], [46], [50], [61], [62], [63], [64], [64], [71], [73], [74], [82], [86] have extended epidemic-like models by considering different infection rates dependent on the types of the layers.

A recent contribution in the context of multiplex networks [87] proposed a generalized epidemic mean-field (GEMF) model capable of Modeling epidemic-like spreading processes with more complex states in multiplex network layers (compared to two or three states in the SIS and SIR models).

3.1.2   Decision-Based Models

Decision-based models (also called threshold models) are based on the idea that an agent may decide to adopt a particular behavior depending on the behavior of its neighbors [88], [89], [90], [91]. For example, a user can join a demonstration if a suitable fraction of his/her friends decide to participate to the event as well. Although *threshold* models may be the more common name in the physics literature, we use *decision-based* models [44], to emphasize that decision is an inherent characteristic of these models.

Existing decision-based studies follow two different approaches [44]: (i) *informational* and (ii) *direct-benefit* effects.

Informational effects. In this approach, decision is based on the indirect information about the decisions of others. Granovetter presented the first decision-based model, called Linear Threshold Model (LTM) [88]. In LTM, each node chooses a threshold value $T_{LTM} \in [0, 1]$ and adopts a new behavior if and only if at least a fraction T_{LTM} of its neighbors has already adopted the new behavior. Based on LTM, Watts [89] studied the roles of thresholds and network structure on information diffusion. The Watts threshold model has also been generalized for multiplex networks in [92], [93]. All these extensions lead to the conclusion that

multiplex networks are more likely to produce global adoption cascades than monoplex networks.

Direct-benefit effects. This approach assumes that there are direct payoffs from copying the decisions of others [94]. Therefore, game-theoretic modeling is at the center of this type of decision-based models. In [95] the authors generalize the model of networked coordination games [91] for spreading processes on multiplex networks. Given a payoff matrix for choosing two possible behaviors A and B, each node is playing a game with its neighbors across all layers. In each round, all nodes update their strategies based on the whole payoff (i.e., the sum of all the payoffs collected in all layers). In [95], the authors derive a lower bound for the success of a new behavior, defined as the eventual adoption of the new behavior across all nodes in the network.

3.2   Theoretical Approaches for Analyzing Spreading Models in Multilayer Networks

The dynamics of spreading models have been studied using different well established mathematical methods. We now describe some of those analytic approaches.

3.2.1   Generating Function

The generating function technique is widely used in the analysis of stochastic processes [96]. Generating functions can uniquely determine a discrete sequence of numbers, and can be useful for computing probability density functions, moments, limit distributions, and solutions of recursions and linked differential-difference equations [97]. Generating functions have also been used to study branching and percolation processes as two important stochastic processes for modeling spread of epidemics over networks.

The branching process model is a simple framework for modeling epidemics on a network [44]. Suppose that an infected agent may come in contact with k other agents while it is infectious, and can spread the disease to each of those with probability p. Each of those k agents (*first wave*) can then get in contact with k other agents, so that the disease could be spread to k^2 individuals (*second wave*), and so on. Questions like whether the process dies out after a set of infection waves or continues indefinitely are of significant interest in the analysis of this process. A theoretical framework for branching processes in multiplex networks has been recently developed [62].

Branching processes, however, can not be applied in situations when the probability to transmit a disease depends on the past history of the destination agent, e.g., if it has already been infected and become immune as in the SIR model [98]. The steady-state behavior of the SIR model can be analyzed by mapping this process into a bond percolation process on graphs [5], [99], [100], and then using existing results for graph percolation [101].

Percolation theory studies the structure of connected clusters in random graphs. It has been shown that there exists a critical probability p_c such that for $p > p_c$ the random graph has a giant connected component (GCC). A percolation transition occurs at the critical occupation probability p_c, which is the point of appearance/disappearance of a GCC. In [102] the authors extends percolation theory to multiplex networks by introducing the concept of *weak bootstrap percolation*

and *weak pruning percolation*. The authors show that these two models are distinct and give origin to different critical behaviors on the emergence of critical transitions, unlike their equivalence in the case of single layer.

3.2.2 Markov-Chain Approximation (MMA)

The Microscopic Markov-Chain Approximation is an established approach to study the microscopic behavior of epidemic dynamics, e.g., the probability that a given node will be infected [6], [76]. This approach can further be categorized as (i) Discrete-time version [103], and (ii) Continuous-time version [7]. In a discrete-time MMA framework, [65] study the malware propagation on a multiplex network where each node in all layers are in same state however, the spreading process is totally independent on each layer. The results show that the dynamics of a SIS contagion process in multiplex networks are equivalent to the spreading in a single layer which is governed by an effective contagion matrix. This allows us to treat epidemic spreading as in a single network. The authors observed that coupling of layers helps the viruses propagation. Moreover, in [104] the authors study epidemic spreading in multiplex networks by using a combination of discrete-time and continuous-time MMA approaches. More in the context of Markov-Chain approximation, the authors in [68], [69] study the spreading of two interacting processes in an arbitrary multiplex network by approximating the spreading process as a discrete-time non-linear dynamical system.

3.2.3 Mean-Field Theory

Markovian modeling is a common approach for modeling stochastic processes between nodes, or in more technical sense, interacting agents in a network. Unfortunately, large Markovian models may become intractable; mean-field theory studies the behavior of such large and complex models by considering a simpler model. Instead of computing the effect of all agents, mean-field theoretic approaches consider a small averaged effect and an external field, replacing the interaction of all other agents. Mean-field theory has been used to capture the macroscopic behavior of the epidemic dynamics such as epidemic threshold and infection size of epidemic-like models [3]. This theory has been widely applied to epidemic processes in monoplex networks, under different assumptions and settings [3]. Some recent works use mean-field approximation for analyzing epidemic-like models in multilayer networks [73], [87].

In [73] the authors determine that the SIS epidemic threshold in an interconnected network with two layers is smaller than the epidemic thresholds of the two networks separately even when the epidemics can not propagate on each network separately and the number of coupling connections is small; the same result may apply to the SIR model. In [87] the authors analyze a generalization of the epidemic-like models for multilayer networks. Mean-field approximation allows the description of the model with a number of nonlinear differential equations with linearly growing state space.

3.2.4 Game Theory

Some researchers have analyzed spreading processes using game-theoretical framework in monoplex as well multilayer settings. Game theory allows modeling the user's behavior

to understand the effect of cooperation and competition on information dissemination. For example, the model proposed in [105] explicitly represents feature of each spreading agent such as reputation and desire of popularity, in addition to the usual structure of the network. The model shows that the emergence of social networks can be explained in terms of maximization of the game-theoretical payoff.

Similarly, the information diffusion model described in [106] takes into consideration various factors pertaining to humans, such as knowledge and belief persuasion, and shows that the speed of spreading is influenced by the features of each individual in the network.

Apart from social networks, studies have also been conducted to understand the information propagation in other settings such as vehicular networks [107]. Recently, game theory has also been studied in multilayer settings. For example, in [95], the authors have studied the diffusion of innovation using the networked coordination game.

4 SPREADING DYNAMICS ON MULTILAYER NETWORKS

The dynamics of spreading processes, e.g., speed or pattern of spreading, are influenced by the properties of underlying multilayer network. In this section we discuss the effect of various properties considered in the literature for interconnected networks (Section 4.1) and multiplex networks (Section 4.2). In Table 4, we summarize and consolidate the discussions.

Aggregating different layers into a single network is one possible way to study multiplex networks [45]. For example, in [108] the authors reduce a multilayer network to a weighted monoplex network, so that the epidemic threshold and infection size of SIR and SIS models on the multiplex networks can be studied by looking at the reduced graph. However, disregarding the inherent multiplex nature of a system could lead to loss of information and wrong conclusions [61]. In this section we will be focused on work that explicitly considers the multiplex nature of the systems.

4.1 Interconnected Networks

The dynamics of spreading processes in interconnected networks can be affected by spectral properties of the combinatorial supra-Laplacian of underlying graph [79], [109], [110]. This matrix and consequently its properties are strongly affected by inter-layer coupling, i.e., coupling (or interaction) strength between layers. In particular, [109] shows that changing the second eigenvalue of algebraic connectivity of an interconnected network has two distinct regimes (layers are decoupled or indistinguishable) and a structural transition phase between them.

Most of the works on spreading processes in interconnected networks studied the impact of inter-layer connections, in terms of *Interaction strength between layers* and *Inter-layer pattern*. Next, we review these works.

4.1.1 Interaction Strength between Layers

We start by describing some measures for the interaction strength, and mention the works which studied their effect on particularly the spreading processes.

Second-nearest neighbors. The expected number κ of neighbors of a node chosen by following an arbitrary link incident

TABLE 4
Summary of the Aspects and Variables Studied in Publications Regarding Spreading Processes in Multilayer Networks

Topics under study	Paper	Underlying Multilayer Network			Spreading Properties		Theoretical Approach
		Type	Intra-layer	Inter-layer	Model	Measures	
Interaction strength	[46]	I	DD	DD	SIR	ETh, Sur	Gn
	[112]	I	DD	DD	SIS	ETh	MF
	[71]	I	Real (AS), Syn (ER-AS, SF-AS)	Syn (R, LL, LH, HH)	SIR	IS, TE	-
Inter-layer pattern	[14]	I	DD, Syn (ER-SF)	DD, Syn (R)	SIS	ETh	MF
	[71]	I	Real (AS), Syn (ER-AS, SF-AS)	Syn (R, LL, LH, HH)	SIR	IS, TE	-
	[73]	I	DD, Syn (ER-ER)	DD, Syn (R, Corr)	SIS	ETh, TE	MF
Multidimensional epidemic thr.	[114]	I	DD, Syn (ER-ER, SF-SF)	DD, Syn (R)	SIR	TE, ETh	Gn
Intra-layer Structure	[63]	M	DD, Syn (ER-SF)	-	SIR	IS, ETh	Gn
Inter-layer Similarity	[64]	M	DD, Syn (ER-ER, ER-SF, SF-SF)	-	SIR	IS, ETh	Gn
	[82]	M	DD	-	SIR	IS	MF
Partially interconnected	[72]	M	DD, Syn(ER-ER,SF-SF)	Par	SIR	IS, ETh	Gn
	[62]	M	DD, Syn(ER-ER,SF-SF)	Par	SIR	IS, ETh	Gn
Layer-switching Cost	[50]	M	DD, Syn(ER-ER)	-	SIR	IS, ETh	MF
	[61]	M	DD, Real(Twitter)	-	SIR	ETh	MC
Spreading velocity	[49]	M	Syn(ER-ER, SW-SW)	-	Watts	CV, CS	-
Interacting Processes	[86]	M	DD	-	SIR	ETh	Gn
	[82]	M	DD	-	SIR	IS	MF
	[74]	M	Syn (ER-ER, SF-SF)	-	SIS	ETh, TE	MF
	[70]	M	Real	-	ICM	IS	-
	[68], [69]	M	Syn (ER-SF), Real	-	SI1I2S	ETh	MC
	[78]	M	DD, Syn (ER-SF)	-	SI1I2S	IS, STh, DTh	MF
Diffusion of Innovations	[92]	M	DD, Syn (ER-SF)	-	Watts	IS	Gn
	[93]	M	DD, Syn (ER-ER)	-	Watts	IS	Gn
	[122]	M	DD, Real	-	Watts	IS	Gn
	[95]	M	Syn (ER-ER)	-	Game	IS	Gm
Effect of resource constraints	[123]	M	Syn (ER-ER,BA-BA)	-	SIR	Eth, IS	-

First column: Topics under study; "thr": threshold. Second column: Reference. Third column: Type of underlying multilayer network; "I": Interconnected network, "M": Multiplex network. Fourth and Fifth columns: intra- and inter-layer connections between layers in underlying multilayer network; "-": Node sets are identical in different layers, "Par": A fraction of nodes are present in all layers with some probability, "Syn": Synthetic dataset, "Real": Real dataset, "DD": Degree-Driven network, "SF": Scale-Free network, "ER": Erdos-Reyni network, "SW": Small-World network, "R": Random inter-connection, "AS": Autonomous Systems, "Corr": Inter-layer connections with different correlation, "LL": Interconnections between lowest-degree nodes of the two layers, "LH": Interconnections between lowest-degree nodes in one layer to highest-degree nodes in other layer, "HH": Interconnections between highest-degree nodes of the two layers. Sixth column: Spreading models. Seventh column: Measures for analyzing spreading properties; "ETh": Epidemic Threshold, "IS": Infection Size, "TE": Temporal behavior of the prevalence, "Sur": Survival probability, "CV": Cascade Velocity, "CS": Cascade Size, "STh": Survival threshold, "DTh": Absolute-dominance threshold. Eighth column: Theoretical Approach; "Gn": Generating function, "MC": Microscopic Markov-Chain approximation, "MF": Mean-Field theory, "Gm": Game Theory.

to a given source can be computed as $\kappa = \langle k^2 \rangle / \langle k \rangle$ where $\langle k^2 \rangle$ and $\langle k \rangle$ are the second and first moment of the node degree distribution, respectively [111]. This measure considered in [46] as a measure for coupling strength. In particular, the authors define an interdependent network to be strongly-coupled if κ_T is larger than both κ_A and κ_B, where κ_A and κ_B are calculated over the individual layers A and B, and κ_T is calculated over the entire coupled network (i.e., including intra- and inter-layer links). On the other hand, a network is defined to be weakly-coupled if $\kappa_B > \kappa_T$ and $\kappa_T > \kappa_A$. The authors show that in the case of a spreading disease (modeled by the SIR model) over a strongly-coupled network, all networks are either in epidemic state or disease free (with the presence of inter-layer links enhancing epidemic spreading). However, in the weakly-coupled case a new mixed phase can exist, with the boundaries dependent on the values of T and $\langle k_{AB} \rangle$, which denote the transmissibility of the SIR model and the average of inter-layer degrees, respectively. In this mixed phase, the disease is

epidemic on only one layer, and not in other layers. Moreover, increasing the inter-layer links only affects epidemic spreading on the layer with more intra-layer links, and the epidemic on the layer with lower number of intra-layer links remain unchanged.

Interconnection topology measure. In [112], the authors propose a purely topological and quantitative measure to distinguish strongly-coupled and weakly-coupled cases in an arbitrary interconnected networks. For an interconnected network with two layers G_1 and G_2, let A_{11} and A_{22} be the corresponding adjacency matrices, and let A_{12} denote the connections between the layers. For this network, the coupling $\Omega(G_1, G_2)$ of the two layers is computed as $\Omega(G_1, G_2) = \frac{\alpha^2 \|A_{12}^T x_1\|_2^2}{\lambda_1(A_{11})\lambda_1(A_{22})}$, where α represents the heterogeneity of intra- and inter-layer connections, and x_1 is the eigenvector of A_{11} belonging to $\lambda_1(A_{11})$. In this measure, larger Ω means stronger coupling.

Inter-layer link density. Inter-layer link density d can be defined as the ratio of the existing inter-layer links m between two layers A and B to the total number of possible such links, giving $d = m/(n_A \times n_B)$, where n_A and n_B are the number of nodes in layer A and B, respectively, of an undirected interconnected network. The maximal inter-layer link density of a completely interconnected network is 1. This interaction strength measure is used in [71] to study the effects of inter-layer links on information spreading, modeled with SIR, in two-layer interconnected networks. The authors find that having more inter-layer links steadily leads to a much larger infection size. In addition, their results show that infection peak happens in two networks at different time, when two networks are sparsely interconnected and the spreading rate is high enough.

4.1.2 Inter-Layer Pattern

The effect of inter-layer pattern (i.e., how the nodes in different layers connect to each other) on the dynamics of spreading processes in interconnected networks has been studied in some recent work. In [113], the authors introduce two quantitative metrics (called *Inter degree-degree correlation* and *Inter-clustering coefficient*) to measure non-random coupling pattern between nodes in interconnected networks. Recently, a simulation-based study in [71] has shown that the inter-layer connections based on the node degree (e.g., interconnections between lowest-degree nodes of the two layers or lowest-degree nodes in one layer to highest-degree nodes in other layer) have less significant impacts on the infection size than the density of interconnections.

Related to this research area, in [14] the authors observe that the epidemic threshold of the SIS model in a two-layer interconnected network is $1/\lambda_1(M + \alpha N)$ where the denominator presents the largest eigenvalue of the matrix $(M + \alpha N)$, α being a real constant for controlling the infection rate between layers, M being a $2n \times 2n$ matrix composed of the adjacency matrix of each layer with size n, and N being a $2n \times 2n$ matrix that represents the inter-layer links between layers. Then, they show that $\lambda_1(M + \alpha N)$ tends to be higher (i.e., a smaller epidemic threshold) if the two nodes u and v with a larger eigenvector component product $x_u y_v$ are connected [14].

On similar lines, the effect of correlations between intra-layer and inter-layer degrees is studied in [73]. In an interconnected network with two layers A and B, let k_{AA} and k_{AB} (resp. k_{BB} and k_{BA}) be the number of a node intra-layer links and inter-layer links. Then, the correlations between intra- and inter-layer degrees can be measured by factors $\langle k_{AA} k_{AB} \rangle$ and $\langle k_{BA} k_{BB} \rangle$. The authors address three different inter-layer patterns based on this type of correlation: (i) random coupling, (ii) linear correlations, and (iii) superlinear correlations. Their results show that if this correlation is strong enough, the outbreak state may arise even if the epidemic threshold is not satisfied in any of the two networks separately.

4.1.3 Multidimensional Epidemic Threshold

When different infection rates are considered for inter and intra-layer edges, a single epidemic threshold for all networks cannot provide an accurate description of real spreading processes in multilayer networks. In [114], the authors study the relation between epidemic threshold and infection rates in the general case of interdependent networks with different infection rates, and introduce a new concept of multidimensional epidemic threshold. They show—both analytically and using simulation—the conditions for multilayer epidemics, i.e., the appearance of a giant connected component spanning all networks.

4.2 Multiplex Networks

Various topics about spreading processes have been addressed in multiplex networks. Here we review some of the most relevant works. Sections 4.2.1, 4.2.2, 4.2.3, and 4.2.4 are mostly focused on the role of network structure, while 4.2.5, 4.2.6, 4.2.7, and 4.2.8 are about the properties of spreading processes.

4.2.1 Intra-Layer Structure

Epidemic dynamics depend not only on how the links are distributed between layers, but also across the same layer [63], [115]. In [63], the authors address information diffusion in a social-physical multiplex network where the information could spread between individuals either through physical or online social networks. They address the effect of clique structures in physical networks (i.e., groups of people who are close to each other) on the epidemic threshold and infection size. In their analytic study, based on heterogeneous bond percolation [116], they show that in large size cliques information spreads faster. To this end, they define three types of link (i) Type-0 (intra-clique) links in the physical layer, (ii) Type-1 (inter-clique) links in the physical layer, and (iii) Type-2 (online) links. Let d_w and d_f denote the numbers of type-1 and type-2 links of a node, respectively. The, with high probability there exists an epidemic state in the entire network when $\sigma = \frac{1}{2}(a_{11} + a_{22} + \sqrt{(a_{11} - a_{22})^2 + 4a_{12}a_{21}}) > 1$, where $a_{11} = \mathrm{E}[(d_w)^2]/\mathrm{E}[d_w] - 1$, $a_{12} = \mathrm{E}[d_w d_f]/\mathrm{E}[d_w]$, $a_{21} = \mathrm{E}[d_w d_f]/\mathrm{E}[d_f]$ and $a_{22} = \mathrm{E}[(d_f)^2]/\mathrm{E}[d_f] - 1$. They also observe a sharp increase in the percentage of individuals (14 to 80 percent) receiving the message when the average clique size increases from 1 to 2. In [115], the same authors show that a larger size online social network may not lead to outbreak in a social-physical network.

4.2.2 Layer Similarity

One aspect that may influence the spreading behavior in multiplex networks is the similarity (or lack of) between layers. There are two important metrics for measuring the level of inter-layer similarity: degree-degree correlation and average similarity of neighbors. *Degree-degree correlation* describes the correlation of degrees of nodes in different layers [64], [82], analogously to degree correlation in monoplex networks [5]. This type of correlation can be measured by factors $\langle k_{AA} k_{BB} \rangle$, where k_{AA} and k_{BB} are the number of a node's intra-layer links in layer A and B, respectively. *Average similarity of neighbors* is defined as $\alpha = \sum_i K_C(i) / \sum_i K_A(i) + K_B(i) - K_C(i)$, where $K_A(i)$ (respectively $K_B(i)$) is the number of neighbors of node i in layer A (respectively B), and $K_C(i)$ is the number of common neighbors of node i in layers A and B. In [64],

[82], the authors study the impact of average similarity between two layers on both epidemic threshold and infection size. They show that a strong positive degree-degree correlation of nodes in different layers could lead to a low epidemic threshold and a relatively smaller infection size. Interestingly, these measures are not significantly affected by the average similarity of neighbors.

4.2.3   Partially Interconnected Multiplex Networks

In partially interconnected multiplex networks, only a fraction of the nodes are present in all layers [117]. This is somewhat in contrast to the link-overlap concept that denotes the fact that some links are shared between the layers. It has been observed that link-overlap among layers adds robustness to the network [118]. In addition, in an empirical study [119] using a large multilayer real dataset, the authors find out that nodes' behavior might differ in different layers. Recently, to understand the link-overlap between layers, Bianconi proposed a statistical mechanics framework [120]. We now examine the effect of partially interconnected scenarios on spreading processes.

In [72] the authors study SIR dynamics on a two-layer multiplex social-physical network. The first layer represents a physical information network where information spreads through face-to-face communication or direct phone calls; the second layer represents a social network. The authors observe that epidemic diffusion (percolation) can happen in the conjoint network, even if no percolation happens within each individual layer. Moreover, the authors also find that the fraction of nodes that receive an information item is significantly larger in the entire network, compared with the case when the layers are disjoint. They show that social networks with $\lfloor n^m \rfloor$ nodes, where n is the size of the physical network and $0 < m < 1$, have almost no effect in the threshold and the expected size of information epidemics in the conjoint network.

In [62] the authors propose a theoretical framework to study the effect of partially interconnected case on SIR tree-like spreading processes. They show that the epidemic threshold of a multiplex network with two layers A and B depends on both the topology of each layer and the fraction q of nodes present in both the layers. When q approaches zero then the spreading process mostly happens in layer A, while when q approaches 1 then the spreading process happens on the fully shared multiplex network. Assuming that an infection is started from a randomly chosen node in layer A, the authors observe that in the limit $q \to 0$ the epidemic threshold of the whole network is $T_c = 1/(\beta_A - 1)$ where β_A is the branching factor of layer A. On the other hand, when $q \to 1$ the epidemic threshold becomes $T_c = 1/\sqrt{[\beta_A - \beta_B]^2 + 4\langle \beta_A \rangle \langle \beta_B \rangle}$. This result implies that the presence of shared nodes lets the epidemic threshold of the layer with the lower propagating capability affect the threshold of the other layer.

4.2.4   Layer-Switching Cost

In some systems modeled as multiplex networks, the diffusion of a process from one layer to another may involve non-zero layer-switching cost or overhead. For example,

retweeting a tweet in Twitter may be more likely than sharing it over other online media (e.g., Facebook) because of the additional effort required in switching the communication channel (cost overhead) [50]. As another example, in the transportation network of a city where the same locations can be part of both subway and bus networks, one can consider the layer-switching cost to move from the subway lines to the bus route. This cost can be both financial or can represent the time required to physically change layers [121].

The effect of the layer-switching cost on the spreading processes has been studied in [50], [61]. In a still unpublished report [50] the authors define layer-switching cost by considering the difference between transmissibilities (i.e., effective infection rates λ) in the SIR model for intra and inter-layer links. They show that the epidemic state will appear if the largest eigenvalue Λ of the simplified Jacobian matrix $\mathbf{J} = \begin{pmatrix} T_{11}\gamma_1 & T_{21}\Gamma_1 \\ T_{12}\Gamma_2 & T_{22}\gamma_2 \end{pmatrix}$ is greater than one (i.e., $\Lambda > 1$), where $\gamma_i = (\langle k_i^2 \rangle - \langle k_i \rangle)/\langle k_i \rangle$ ($\langle k_i \rangle$ and $\langle k_i^2 \rangle$ are the first and second moment of the degree distribution of the layer i), $\Gamma_i = \langle k_i k_j \rangle / \langle k_i \rangle$, and T_{ij} is the transmissibility over the link between layer i and j. They show that Λ is a function of the node degrees δz and infection rates $\delta \lambda$, and study their effect on the epidemic threshold. In particular, when both layers have the same average degree the epidemic threshold increases for larger difference between intra- and inter-layer infection rates as it gets more difficult to spread to other layers (high layer crossing overhead). For the constant difference in rates, if the difference of the average degree of the two layers gets larger (i.e., a layer becomes denser), the epidemic threshold decreases as denser layers facilitate spreading. Finally, they find a threshold for the difference of average degrees, above which the epidemic threshold decreases as the difference in rates becomes larger. These results have been obtained on Erdős-Renyi random graphs.

Similar findings were presented in [61], where authors study the SIS model in multiplex networks using a contact-contagion formulation with different infection rates for intra and inter-layers. They observed that the layer with largest eigenvalue controls the epidemic threshold of the entire network.

4.2.5   Spreading Velocity

The presence of multiple layers can impact the speed at which a piece of information can spread through the network; intuitively, one would expect that multiple layers speed up the spreading process since more links are available and nodes can receive more pieces of information from multiple communication channels. This intuition has indeed been confirmed in [34], [72], [79]: the authors show that the coupling of two layers in multiplex networks can lead to speed up a spreading process in the entire network.

However, some empirical studies point out that different link types [124], [125], [126] and topologically inefficient paths [127] may actually decrease the spreading speed in monoplex networks; this suggests that this area of research needs more attention. Recently [49] addressed the velocity of the cascade process in multiplex networks by considering the role of inter-layer links. In this simulation-based study,

the authors find that the obstruction of an inter-layer link connecting the shortest paths distributed in multiple layers leads to a slower spreading process in multiplex networks. In this context, the results in [128] on different types of random walks on multiplex networks have important implications for spreading processes. The authors show that the time required for a random walker to visit the nodes depends on the underlying topology, the strengths of inter-layer links and the type of random walk.

4.2.6 Interacting Spreading Processes

In the real world, many spreading processes may happen at the same time over the same network: for example, multiple diseases may spread concurrently on the same population and produce different cascades [129]. These processes may interact with each other so that the dynamics of one of the diseases may be affected by those of the others. Moreover, depending on the nature of each spreading process, the underlying cascades can differ.

The interaction of different spreading processes on monoplex networks can be addressed in the settings of multiplex networks. A common assumption is that spreading processes can become extinct; in this case, one process will dominate the other one even when the infection rates of both of them are above the epidemic threshold [130]. Recently, in [131] the authors relax this assumption and address the domination time. They find that it depends on the number of infected nodes at the beginning of the domination period.

Other works used a game-theoretical framework to investigate interacting spreading processes. Kostka et al. [132] addressed the spreading of competing rumors in social networks as a strategic game. It has been shown that being the player that starts the game for the rumors is not always an advantage. Compared to this work, [133], [134], [135], [136], [137] have studied the competition between companies who use their resources to maximize the adoption of their product in a social network. There is a subtle difference between these works: [133] uses a stochastic model, whereas [134] uses a deterministic model, and in [135] individuals made rational decisions. In [136] the authors presented a game theoretic model based on local influence process, while in [137] a local quasi linear model is exploited.

An important step towards a theoretical framework for interacting processes was taken in [86]. Extending the bond percolation analysis of two virus spreading processes for a two-layer network [138], the authors addressed the interaction between two SIR processes spreading successively on a multiplex network. They find that cross-immunity (through the interaction between processes) is more effective where high-degree nodes in different layers are connected. However, their analytic approach is static and does not cover the evolution of the system over time. This issue was considered in [82], where authors addressed the interaction between spreading processes on multiplex networks in terms of the heterogeneity level of contact patterns between nodes, various degree correlations and overlapped links between the layers. By considering two interacting processes, the first being an undesirable disease and another being an immunizing process, the authors have shown that

the positive degree correlation increases the efficiency of immunization, while overlap facilitates the invasion of disease. In [74] the authors proposed a framework based on mean field theory to study the spreading of two concurrent processes that allows to derive the epidemic threshold of each process. Moreover, this approach can be extended to various epidemic models (such as SIR, SIS, and SEIR). They found that the epidemic thresholds of both processes depend on the parameters that characterize the underlying network structure and on the dynamics of each process.

Some related works have been proposed in the computer science community. In [70], the authors have studied the problem of limiting misinformation propagation in a social network, called influence limitation. They have extended the independent cascade model to analyze the dynamics of multiple cascades over a multiplex network. Moreover, [68], [69] have studied the spreading of two interacting memes, modeled as SI_1I_2S (an SIS-type model), in an arbitrary two-layer multiplex network. In this model, each node can be infected by virus 1 or 2 (represented as I_1 and I_2, respectively). They show that the meme with larger first eigenvalue will eventually prevail in the entire networks. However, this result is challenged by [78] where the authors study the long-term coexistence of two SI_1I_2S virus spreading process over an arbitrary multiplex network (note that the authors referred this model as SI_1SI_2S). They find that the long-term coexistence of both viruses depends on the structural properties of the underlying multiplex network as well as epidemic-related factors. In particular, they show that the negative correlation of network layers makes it easier for a virus to survive, but the extinction of the other virus is more difficult.

4.2.7 Diffusion of Innovations

Diffusion of an innovation (new behavior, ideas, technology, products) over networks and the role of underlying network in its dynamics has received considerable interest in social sciences and economics [91], [139], [140], [141]. Recently, this problem has been studied in the framework of multiplex networks. [92] studies the condition and size of global spreading cascades of innovations in a multiplex network with multiple types of interactions by using an extension of Watts' threshold model. In particular, they assume that a node becomes infected if the fraction of infected neighbors in any link type is higher than a given threshold. The authors in [93] propose a content-dependent threshold model in which each link type is associated with a relative bias in spreading a given content (e.g., new product). More in this context, the authors of [122] have shown that the existence of a multiplex correlated graph is a condition for sustaining a viral spreading process. To identify the conditions for viral cascading, they map this process to a correlated percolation model. Considering the approach of direct-benefit effects, [95] finds a lower bound for the success of an innovation (i.e. how many people in the network adopt a specific strategy) in a game-theoretic framework.

4.2.8 Resource Constraints

In a realistic scenario, nodes of a multiplex network share limited resources. This can impact the dynamic of spreading processes in such networks; for example, a person shares

her/his time between his/her accounts in different online social networks such as Facebook and Twitter. This is studied in [123] by using a variation of the SIR model called *constrained SIR*. In each step of constrained SIR, there is a maximum value on the number of neighbors that each node can infect. The authors find that, in agreement with previous studies [142], in the absence of resource constraints, positively correlated coupling leads to a lower epidemic threshold than a negative correlation. However, in the presence of constraints, spreading is less efficient in positively correlated coupling than negatively correlated networks.

5   APPLICATIONS

Spreading processes in multilayer networks have a large number of applications, such as understanding the dynamics of cascades [48], [143], maximizing the influence of information in the context of viral marketing [144], or selecting a subset of nodes in a network where to place sensors in order to detect the spreading of a virus or information as quickly as possible [145].

The application areas can be roughly categorized into two classes:

- *Forward prediction*. Applications that need to steer the network into a particular desired state. Viral marketing and influence maximization fall under this category.
- *Backward prediction*. Applications that require to predict how a given piece of information will spread in a network. Outbreak detection, cascade detection and immunization are some examples under this category.

In this section we discuss some applications of spreading processes, representing the two categories above.

5.1   Influence Maximization

Influence maximization has the goal of spreading a particular message as quickly as possible to a large number of nodes. This is usually done by seeding the information through key "strategic" nodes in such a way that they can help in reaching out most of the network. The identification of such strategic nodes is therefore essential to ensure that the message spreads quickly and effectively. The problem of influence maximization in networks has traditionally been focused on finding influential nodes, that is, a (possibly small) subset of nodes that have the maximum influence to spread the message [146], [147], [148].

Recent works in the context of multilayer networks address the problem of identifying influential nodes in various domains, such as ranking scientific authors according to multiple levels of information (e.g., citation networks and co-authorship graphs) [149], studying the spreading of a virus [67] or identifying the most active individuals in microblogging platforms based on multiple types of relationships between individuals [150].

In general, the influential nodes are the top-k nodes according to some centrality measure, such as betweenness centrality [151], [152], eigenvector centrality [153] or page rank [154]. It is important to observe that results for monoplex networks do not always generalize to multilayer networks; as an example, in [155] the authors show that the k-shell index [156] proposed for identifying the influential nodes in monoplex networks loses its effectiveness in interconnected networks, so they introduce a new measure which considers both structural and spreading properties.

It is also possible to look at the problem of information dissemination from a completely different perspective, that is, by looking at the set of possible messages that can be diffused, and find which message is likely to survive longer in the network compared to others. As an example, in [68] the authors propose a new metric to quantitatively assess the probability that a message spreads more than another; therefore, given a set of different but equivalent messages, it is possible to select the one which will likely propagate with a higher fraction of nodes in the multilayer graph.

5.2   Immunization Strategies

How can information dissemination improve the resilience of a population against a spreading disease? To answer this question, various works have investigated the role of information dissemination (or awareness) with respect to the control of a disease spreading over multilayer networks. In [157] the authors consider a two-layer network, where the infection layer (where an epidemic spreads) is a Watts-Strogatz small-world network, and the prevention layer is modeled as a dynamic process in a Barabási-Albert scale-free network. The authors observe that, in this scenario, epidemic waves are strongly reduced to small fluctuations, but in certain situations the prevention layer actually helps the disease to survive. In [158], [159] the authors investigate a SIR model where better-informed nodes have a reduced susceptibility, showing that this can raise the threshold for the widespread spreading of the infection. In a different kind of study, [160] propose the Behavior-Immunity model that allows measurement of vaccination effect based on the impact of proactive immunization strategies. In [161] the authors study a process in which SIS dynamics are coupled with a process that rewires intra-layer edges between susceptible and infected nodes on an interconnected network.

Studies dealing with epidemic spreading are not only based on synthetic networks, but consider real networks as well. For example, in [162] the authors considered information and disease spreading processes together, using mobile-phone dataset. Some researchers have proposed metrics for the control of information awareness to disease propagation. For example, in [163], the authors found a meta-critical point for the epidemic onset leading to disease suppression. This critical point depends on awareness dynamics and the overlay network structure. An additional study from the same authors [164] identifies the relation between the spreading and immunization processes for a wide range of parameters; additionally, in the presence of a *mass-media effect* in which most of the individuals are aware of the infection, the critical point disappears. Epidemic spreading in two-layer networks (one layer spreading a disease and the other diffusing awareness on the infection) is analyzed also in [165]: the authors conclude that the similarity between the two layers allows the infection to be stopped with a sufficiently high precaution level. Interested readers can refer to [166] for additional references for this research area.

In [167], [168] the authors use the SAIS model [169] to find an optimal infection information propagation overlay in an underlying network to improve resilience against epidemic spreading. SAIS is an extension of the traditional SIS model where 'A' represents a new Alert state. The authors prove that the spectral centrality of nodes and links determines such overlay network. They find that controlling the health status of a small subgroup of the nodes and circulating the information has a considerable role in disease prevention. The same authors use this model to address the importance of individuals' responsiveness in the progress of an epidemic [170].

5.3 Epidemic Routing in Delay-Tolerant Networking (DTN)

Delay Tolerant Networking [171] seeks to address the issues arising in heterogeneous networks where individual nodes may lack continuous connectivity. Many useful types of networks fall into this category: for example, a commuter bus equipped with short-range communication capabilities can carry messages from one stop to another. Other examples include deep space communication, where delays can be measured in minutes during which one of the endpoints may have moved out of sight, or sensor networks where communications must be scheduled at specific points in time to preserve power.

Routing and resource discovery on DTNs are more challenging than the equivalent problems on regular communication networks, where link failures are the exception rather than the norm. Traditionally, routing in DTN is achieved using epidemic routing algorithms [172] over a (directed or undirected) graph whose edges represent the current active links. Multiple types of communication channels may be available at the same time: for example, a sensor node could be equipped with both short-range (low power consumption, relatively high bandwidth) and long-range (high power consumption, low bandwidth) RF links that can be jointly described using a two-layer network. Finding the "best" route according to some latency and energy constraints is an important application of forward prediction in multilayer networks.

5.4 Malware Propagation in the Internet

Studying the propagation of malware over the Internet, and possibly designing networks and applications that can slow down and contain malware outbreaks, is an important application of both forward and backward prediction in the context of information dissemination.

Nodes belonging to modern computer networks include mobile devices (smartphones, tabled, portable computers) that are generally equipped with multiple wired and/or wireless communication interfaces. Moreover, applications interact with other applications running on devices that may not be in the immediate neighborhood. Therefore, not only the communication channels define multiple connection layers, but also the interactions of applications should be taken into account as an additional layer.

A piece of malware trying to propagate through the computer network may take advantage of all available physical connections to spread to other devices, and also hijack applications to infect remote nodes. Wang et al. [9] studied the spreading dynamics of a mobile phone virus capable of infecting phones by bluetooth or through MMS messages. This study is actually an interesting example of analysis of malware propagation on a two layer network. A link (u, v) between phones u and v exists on the first layer if and only if u and v are physically close together, so that bluetooth communication is possible. A link (u, v) on the second layer exists if and only if the address book of phone u contains the number of phone v, so that the malware infecting u can try to send a copy of itself to v through MMS.

Obviously, the study can be extended to take into consideration other types of links, and therefore additional layers. Understanding the spreading pattern of malware over multilayer networks can be extremely valuable both for predicting the extension of an infection (forward prediction), and also to understand where countermeasures can be placed in order to contain the epidemic (backward prediction), pretty much in a very similar manner as epidemics among living organisms already described in Section 5.2.

6 CONCLUSION AND OPEN PROBLEMS

Spreading processes in multilayer networks is an active and not yet consolidated research field, and therefore offers many unsolved problems to address. In some cases, phenomena that are quite well understood in monoplex networks are comparatively not well understood in the context of multilayer networks; in other cases, completely novel ideas, algorithms and analysis, specific to multilayer networks have to be developed. Some research directions are illustrated below.

Empirical study of information diffusion. In general, collecting real datasets related to a multilayer network is nontrivial [173]. This issue is even more challenging when one tries to gather data on both the spreading process and the structure of the underlying multilayer network. To the best of our knowledge there are no works based on real datasets on information diffusion in multilayer networks, and the totality of existing works on multilayer spreading are based on simulation or analytic studies. On the other hand, real-world multilayer networks are sometimes large and non-trivially observable, since no single company or institution has full control over all layers. Network sampling strategies [52], [174] can be used to address this issue by decreasing the expense of processing large real networks. Thus, it is worth exploring how different sampling approaches can impact the measurement of spreading processes [53]. In addition, one can explore if there are other ways to infer the structure of the spreading graph, e.g., by injecting suitable messages at given points and track them (*graph tomography*).

Metrics and measurements. Several metrics have been defined for monoplex networks [175], such as diameter, distances, and various centrality metrics. Some of these metrics have been extended to multilayer networks. For details and recent papers in this field, refer to [32, Section 4.2] and [33, Section 2.2]; see also the result of using structural metrics for characterizing a real-world multiplex network [176]. However, it is important to investigate if new metrics, specific to multilayer networks, can be defined. An interesting aspect would be to propose new metrics specific to time-varying phenomena. Another important research direction

would be to explore how these metrics affect the propagation of information. As an example, it would be interesting to measure the correlation between metrics indicating the presence of multilayer communities [177] and the metrics to characterize spreading processes summarized in this paper.

New models for spreading processes in multilayer networks. As already described in this paper, spreading processes in multilayer networks are driven by different mechanisms with respect to the single layer case. The study of diverse topics such as the propagation of opinions about a new product over social networks, or the spreading of a virus across different species (e.g., avian flu spreading through birds and humans), requires the development of suitable spreading models that take into consideration the existence and interactions of different layers within a network. In this paper, we have discussed some existing works on spreading processes in multilayer networks; other phenomena may require novel spreading models to be developed. For example, the *data mining* approach proposed recently in [178] can be considered for Modeling information diffusion in heterogeneous information networks. Another interesting research direction is modeling and analyzing the spreading process on multilayer networks from *game-theoretic* approach. Information diffusion on monoplex network has already been studied from game theory perspective, in which authors postulate an increase in utility for players who adopt the new innovation or learn the new information if enough of their friends have also adopted [91].

Data visualization. An old motto says that "Seeing is Believing". Indeed, many phenomena are first observed, and then suitable models are built to explain the observations. In the context of information diffusion, data visualization tools can provide a first impression of what is going on, and suggest that something worth investigating may be happening indeed. Information diffusion is a dynamic phenomenon, requiring an additional dimension (time) to be visualized [179]. Spreading processes in multilayer networks also require the visualization of different layers, and it is not yet clear what is the most effective and understandable way to provide this kind of information. So far, only a few works have addressed the problem of multilayer network visualization [180], [181]. As a recent contribution in this direction, in [180] the authors introduce a methodology for the analysis and visualization of multilayer networks implemented in an open-source software called *muxViz*.

Time-varying networks. Many real-world networks exhibit a mutable structure, meaning that nodes and links change over time [182]. The spreading processes on such time-varying (monoplex) networks is addressed in recent works [183], [184], [185], [186]. Indeed, both types of dynamics (i.e., dynamics of spreading processes and dynamics of underlying networks) are considered in this field. However, studying this problem in time-varying multilayer networks is more difficult [187]. Recently, in a still unpublished report [188] the authors utilize the mathematical formulation of multilayer networks proposed in [45] to study spreading processes on time-varying networks.

Evolution of underlying network structure and spreading process. The coevolution of spreading processes and underlying structures in adaptive (monoplex) networks network, where nodes change their neighborhood as a response to receiving new information, have been considered in [82], [189], [190], [191]. An interesting observation is that changing the underlying network, e.g., by reducing or modifying contacts to prevent infection, does not always lead to reduction of spreading [192]. This problem becomes more complex if the underlying network is modeled as a multilayer network, requiring further research.

Outbreak detection. Outbreak detection is a technique for the detection of spreading of a virus (or information) in a network as quickly as possible [145]. The problem of outbreak detection is worth exploring in the area of multilayer networks.

ACKNOWLEDGMENTS

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